Review for Chapter 8 Individual Test Name: Yuy

## Lesson 8.1.1 Limits at Infinity

1. Identify the dominant term in each of the following expressions.

a. $5^x - 4x^6 + 2x^2 - 6^x$ b. $45x^2 - 27x + \log x + \sqrt{x}$ c. $\sqrt{x^6} - 5x^2$ $x^3$

2. Evaluate the following limits.

a. $\lim_{x\to\infty} 12x^3 - 1.2^x$ $\boxed{-\infty}$ b. $\lim_{x\to\infty} 3500(4^x) - 5^x$ $\boxed{-\infty}$ c. $\lim_{x\to\infty} 15\sqrt{x} - x$ $\boxed{-\infty}$ d. $\lim_{x\to\infty} -3x^2 + 42\log x$ $\boxed{-\infty}$

## Lesson 8.1.2 Limits of Rational Functions (at infinity)

1. Evaluate the given limits.

a. $\lim_{x\to-\infty} \frac{(x^2-1)(x^2+1)}{(2x^2-3)(2x^2+5)}$ $\frac{x^4}{4x^4}$ $\frac{1}{4}$ $\boxed{\frac{1}{4}}$ b. $\lim_{x\to-\infty} \frac{3x^2-x^5+2}{5x^2+4x^4+1}$ $\frac{-x^5}{4x^4} \to \frac{-x}{4}$ $\boxed{+\infty}$ c. $\lim_{x\to-\infty} \frac{3x^4+x^2}{(x+1)^3}$ $\frac{3x^4}{x^3} \to 3x \to \boxed{-\infty}$

d. $\lim_{x\to\infty} \frac{7x^3+x}{(2x-1)(3x^2+1)}$ $\frac{7x^3}{6x^3}$ $\boxed{\frac{7}{6}}$ e. $\lim_{x\to\infty} \frac{x^2+4}{x^3+x-1}$ $\frac{x^2}{x^3} \to \frac{1}{x}$ $\boxed{0}$ f. $\lim_{x\to-\infty} \frac{\sqrt{4x^4-3x+1}}{(2+3x)^2}$ $\frac{|2x^2|}{9x^2} \to \frac{2}{9}$ $\boxed{\frac{2}{9}}$

## Lesson 8.1.3 Rational Functions

1. Evaluate the given limits.

a. $\lim_{x\to5} \frac{x^2-25}{x-5}$ $\frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}} = \boxed{10}$ b. $\lim_{x\to1} \frac{x-1}{x^2-1}$ $\frac{\cancel{x-1}}{(x+1)\cancel{(x-1)}}$ $\boxed{\frac{1}{2}}$ c. $\lim_{x\to-3} \frac{x^2-9}{2x^2+7x+3}$ $\frac{\cancel{(x+3)}(x-3)}{(2x+1)\cancel{(x+3)}} = \frac{-6}{-5} = \boxed{\frac{6}{5}}$

2. Write a possible rational function with the following characteristics.

a. Hole at x = 2, Vertical Asymptote at x = -3 $\boxed{f(x) = \frac{x-2}{(x-2)(x+3)}}$ b. Horizontal Asymptote at y = 0, Vertical Asymptotoe at x = 0 $\boxed{f(x) = \frac{1}{x}}$

c. Vertical Asymptote at x = 4, Zero (x-int) at x = -2 $\boxed{f(x) = \frac{(x+2)}{(x-4)}}$ d. Hole at x = -1, Horizontal Asymptote at y = 2 $\boxed{f(x) = \frac{2(x+1)}{(x+1)}}$

## Lesson 8.2.1 The Number *e*

1. Evaluate each expression without using a calculator.

a. $\ln e^4$ $\boxed{4}$ b. $\ln\left(\frac{1}{e}\right)$ $\boxed{-1}$ c. $e^{\ln\sqrt{7}}$ $\boxed{\sqrt{7}}$ $\ln_e x = \ln_e \sqrt{7}$ d. $\ln e^x - \ln e^x$ $= \boxed{0}$ $x - x$ $\ln\frac{e^x}{e^x}$ $\ln 1$ $e^{??} = 1$ $= 0$

2. Solve each expression for x. Leave exact answers.

a. $\ln(2x+3) = 29$ $e^{29} = 2x+3$ $\boxed{\frac{e^{29}-3}{2} = x}$ b. $\ln(4) + \ln(3x) = 1$ $\ln(12x) = 1$ $e^1 = 12x$ $\boxed{\frac{e}{12} = x}$ c. $\log_2(\log_3 x) = 4$ $16 = \log_3 x$ $2^4 = \log_3 x$ $\boxed{3^{16} = x}$

d. $3e^{4x} - 1 = 35$ $3e^{4x} = 36$ $e^{4x} = 12$ $4x/4 = \ln 12/4$ $\boxed{x = \frac{\ln 12}{4}}$ e. $\frac{2e^{x-6}}{2} = \frac{100}{2}$ $e^{x-6} = 50$ $\ln e^{x-6} = \ln 50$ $x - 6 = \ln 50$ $\boxed{x = \ln 50 + 6}$ f. $4^{(-3x+4)} = 128$ $(2^2)^{(-3x+4)} = 2^7$ $-6x + 8 = 7$ $\boxed{x = 1/6}$

## Lesson 8.2.2 Applications of *e*

$A = 500000\left(1+\frac{.04}{12}\right)^{12\cdot 20} = \$1111291.04$

$A = 500000e^{.04(20)} = \$1112770.46$

1. Suppose $500,000 was invested with a 4% annual interest rate for a period of 20 years. What difference would there be in the final value of the investment if the interest was invested continuously instead of monthly?

$\boxed{\$1479.42 \text{ more if continuously}}$

2. $3,000 was invested for 8 years with the interest compounded continuously. If the investment is worth $7,700 at the end of 8 years, what was the interest rate?

$\frac{7700}{3000} = \frac{3000e^{8r}}{3000}$   $2.567 = e^{8r}$   $\ln 2.567 = \ln e^{8r}$   $\frac{\ln 2.567}{8} = \frac{8r}{8}$   $r \approx .1178$   $\boxed{11.78\%}$

3. How long will it take any investment to quadruple if it earns 5.6% interest, compounded continuously?

$\frac{4P}{P} = \frac{Pe^{.056t}}{P}$   $4 = e^{.056t}$   $\ln 4 = \ln e^{.056t}$   $\frac{\ln 4}{.056} = \frac{.056t}{.056}$   $\boxed{t \approx 24.76 \text{ yrs.}}$

4. A 32 gram sample of radioactive iodine decays in such a way that the mass $m$ (grams) remaining after $t$ days is given by the function $m(t) = 32e^{-0.089t}$. After how many days are there only 16 grams remaining?

$\frac{16}{32} = \frac{32e^{-.089t}}{32}$   $\frac{1}{2} = e^{-.089t}$   $\ln\frac{1}{2} = \ln e^{-.089}$   $\frac{\ln\frac{1}{2}}{-.089} = \frac{-.089t}{-.089}$   $\boxed{t \approx 7.79 \text{ days}}$

5. Tommy deposits $250 into an account for 6 years. How much interest is earned if the account offers 4.2% annual interest compounded quarterly?

$A = 250\left(1+\frac{.042}{4}\right)^{4\cdot 6} = \boxed{\$321.23}$   $\boxed{\text{Interest} = \$71.23}$

## Lesson 8.2.3 Infinite Geometric Series

1. Find the given sum, or explain why it does not exist.

$r = \frac{-2/3}{1} = -2/3$    $3 + -1.2 + .48$   $r = \frac{-1.2}{3} = -.4$    $r = \frac{2/5}{3/5}$   $\frac{4/15}{2/5} = \frac{4}{6} = \frac{2}{3}$

a. $1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \ldots$

$r = -\frac{2}{3}$   $\frac{1}{1 - -2/3} = \frac{1}{5/3} = \boxed{\frac{3}{5}}$

b. $\sum_{p=0}^{\infty} 3(-.4)^p$

$r = -.4$   $\frac{3}{1 - -.4} = \frac{3}{1.4} \approx \boxed{2.14}$

c. $\frac{3}{5} + \frac{2}{5} + \frac{4}{15} + \frac{8}{45} + \ldots$

$\frac{3/5}{1 - 2/3} = \frac{3/5}{1/3} = \frac{3}{5}\cdot\frac{3}{1} = \boxed{\frac{9}{5}}$

d. $4^7 - 4^5 + 4^3 - 4 + \ldots$

$\frac{-4^5}{4^7} = -.0625$   $\frac{4^7}{1 - -.0625}$   $\boxed{15420.23529}$

e. $\sum_{i=1}^{\infty} 6\left(\frac{2}{3}\right)^i$   $4 + \frac{8}{3} + \frac{16}{9}$

$r = \frac{8/3}{4} = \frac{2}{3}$   $\frac{4}{1 - \frac{2}{3}} = \frac{4}{\frac{1}{3}} = \boxed{12}$

f. $2.5 + 1.75 + 1.225 + 0.8575 + \ldots$   $r = \frac{1.75}{2.5} = .7$

$r = .7$   $\frac{2.5}{1 - .7} = \frac{2.5}{.3} = \boxed{8\frac{1}{3}}$

**Review Topic: Evaluating functions.**

Evaluate each given the functions: $f(x) = 3x^2 - x$ and $g(x) = (x-3)^2 + 4$

a. $f(-2)$

$3(-2)^2 - (-2)$
$3(4) + 2 = 14$

b. $g(7)$

$(7-3)^2 + 4$
$16 + 4 = 20$

c. $f(x-2)$

$3(x-2)^2 - (x-2)$
$3(x^2 - 4x + 4) - x + 2$
$3x^2 - 12x + 12 - x + 2$
$3x^2 - 13x + 14$

d. *find x if* $g(x) = 20$

$20 = (x-3)^2 + 4$
$16 = (x-3)^2$
$\pm 4 = x - 3$
$4 = x - 3$   $-4 = x - 3$
$7 = x$ or $x = -1$

**Review Topic: Writing equations of lines in point slope form.**

a. *Through* $(1, -20)$ *and* $(-14, 32)$

$m = \frac{-20-32}{1--14} = \frac{-52}{15}$

$\boxed{y = -\frac{52}{15}(x-1) - 20}$

b. *Parallel to* $y = \frac{3}{2}x - 4$ *through* $(-2, -7)$

$\boxed{y = \frac{3}{2}(x+2) - 7}$

c. *Passes through* $(-31, 12)$ *and is perpendicular to the line that goes through* $(2, 7)$ *and* $(-1, 5)$.

$m = \frac{7-5}{2--1} = \frac{2}{3}$   $\perp m = -\frac{3}{2}$

$\boxed{y = -\frac{3}{2}(x+31) + 12}$