Calculus 3 **Name** CK

Review – Solving First Order Differential Equations

(After 2.1)

1. Find the general solution for $y^2 y' = (1-x)$.

$$y^2\,dy = (1-x)\,dx$$

$$\frac{1}{3}y^3 = x - \frac{1}{2}x^2 + c_1$$

$$y^3 = 3x - \frac{3}{2}x^2 + c$$

$$\boxed{y = \sqrt[3]{3x - \frac{3}{2}x^2 + c}}$$

2. Find the particular solution for $y'\sin(2y) - \cos x = 0$; $x = \frac{\pi}{2}$ when $y = 0$.

$$\sin(2y)\,dy = \cos x\,dx$$

$$-\frac{1}{2}\cos(2y) = \sin x + c_1$$

$$\cos(2y) = -2\sin x + c$$

$$1 = -2 + c \rightarrow c = 3$$

$$\boxed{\cos(2y) = 3 - 2\sin x} \qquad y = \frac{\cos^{-1}(3-2\sin x)}{2}$$

3. Find the particular solution for $2e^{2x} - y = xy'$; $x = 1$ when $y = 6$.

$$xy' + y = 2e^{2x}$$

$$\frac{d}{dx}(xy) = 2e^{2x}$$

$$xy = e^{2x} + c$$

$$6 = e^2 + c \rightarrow c = 6 - e^2$$

$$xy = e^{2x} - e^2 + 6$$

$$\boxed{y = \frac{e^{2x} - e^2 + 6}{x}}$$

4. Find the particular solution for $3x^2y' = 5x - 6xy$; $x = 2$ when $y = 1$.

$$3x^2y' + 6xy = 5x$$

$$\frac{d}{dx}\left(3x^2y\right) = 5x$$

$$3x^2y = \frac{5}{2}x^2 + C$$

$$3(4)(1) = \frac{5}{2}(4) + C \Rightarrow C = 2$$

$$3x^2y = \frac{5}{2}x^2 + 2$$

$$\boxed{y = \frac{5}{6} + \frac{2}{3x^2}}$$

5. (challenge) Find the general solution for $y = \left(3y^3 + x\right)\frac{dy}{dx}$.

$$y = 3y^3y' + xy'$$

quotient rule! $\left\{ \dfrac{y - xy'}{y^2} = \dfrac{3y^3y'}{y^2} \right.$

$$\frac{d}{dx}\left(\frac{x}{y}\right) = 3yy'$$

$$\frac{x}{y} = \frac{3}{2}y^2 + C \quad \text{or} \quad \boxed{\frac{x}{y} - \frac{3y^2}{2} = C}$$