AP Calculus BC   Name_________________________________

Summer Preparation

Welcome to AP Calculus BC! This packet is a set of problems that you should be able to do before entering this course. Many of them are algebra or trigonometry skills. Some are calculus skills you learned in Honors Precalculus. Some are easy but some are quite challenging. You may print this and do the work here (if your writing is relatively small and neat) or show all your work on computer paper, lined paper, or graph paper (if your writing is large and/or sloppy). The expectation is that you will not use a calculator for any of the work. (You can of course check your work with the calculator). You will submit your work via Google Classroom before the first day of school. DO NOT WAIT UNTIL THE LAST MINUTE. This will take some time! The first test will include material from this packet and will be without the use of a calculator.

There is an answer key at the end of the problems. If you believe there are any mistakes, please email me at nancycsarny@solonboe.org. If you struggle with some of the concepts, you may look online for video help, or you may request a Google Meet with me (just email me).

I am looking forward to working with you next year!

Mrs. Csarny

NOTE TO STUDENTS COMING FROM AP CALCULUS AB: You may NOT use l'Hôpital's rule to evaluate any limits in this packet!

1. Evaluate the following limits given $f(x)=\dfrac{x^3-9x}{x^2-7x+12}$.

   A. $\lim\limits_{x\to 0} f(x)=$

   B. $\lim\limits_{x\to 3} f(x)=$

   C. $\lim\limits_{x\to 4^+} f(x)=$

   D. $\lim\limits_{x\to 4^-} f(x)=$

   E. $\lim\limits_{x\to 4} f(x)=$

   F. $\lim\limits_{x\to \infty} f(x)=$

   G. Write the coordinates of any holes in the graph of $f(x)$:

   H. Write the equations of any vertical asymptotes or horizontal asymptotes of the graph of $f(x)$:

2. Rewrite $g(x)=\dfrac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$ as a rational function, then evaluate the following:

   A. $\lim\limits_{x\to 5} g(x)=$

   B. $\lim\limits_{x\to 0} g(x)=$

   C. $\lim\limits_{x\to \infty} g(x)=$

   D. Write the coordinates of any holes in the graph of $g(x)$:

   E. Write the equations of any vertical asymptotes or horizontal asymptotes of the graph of $g(x)$:

3. Rewrite $h(x)=\dfrac{9-x^{-2}}{3-x^{-1}}$ as a rational function, then evaluate the following:

   A. $\lim\limits_{x\to \frac{1}{3}} h(x)=$

   B. $\lim\limits_{x\to 0^+} h(x)=$

   C. $\lim\limits_{x\to 0^-} h(x)=$

   D. $\lim\limits_{x\to 0} h(x)=$

   E. $\lim\limits_{x\to \infty} h(x)=$

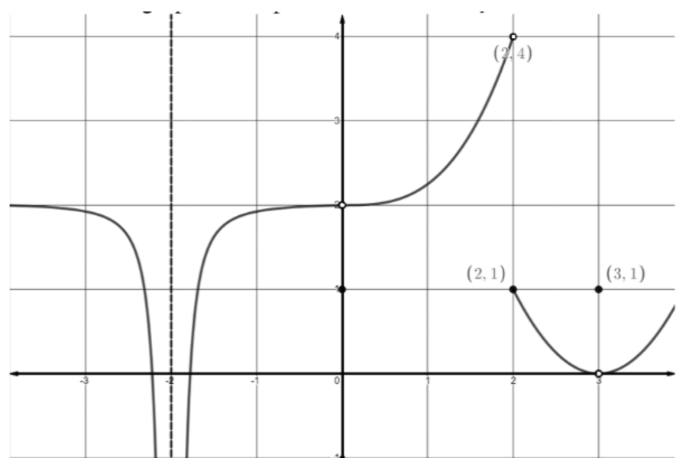
   F. Write the coordinates of any holes in the graph of $h(x)$:

   G. Write the equations of any vertical asymptotes or horizontal asymptotes of the graph of $h(x)$:

4. Use the rationalizing technique to evaluate $\lim\limits_{x\to 0}\dfrac{\sqrt{x+1}-1}{x}$

5. Find the instantaneous rate of change of $f(x) = \frac{1}{x}$ by evaluating $\lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$.

6. Use the graph of $f(x)$ to evaluate the following:



A. $\lim_{x \to -2} f(x) =$

B. $\lim_{x \to 0} f(x) =$

C. $f(0) =$

D. $\lim_{x \to 2^-} f(x) =$

E. $\lim_{x \to 2^+} f(x) =$

F. $\lim_{x \to 2} f(x) =$

G. $\lim_{x \to 3} f(x) =$

7. Rewrite each expression in the form $ca^p b^q$ where $c,\ p,$ and q are numbers.

A. $\frac{a(2/b)}{3/a}$

B. $\frac{a^{-1}}{(b^{-1})\sqrt{a}}$

C. $\frac{ab - a}{b^2 - b}$

D. $\left(\frac{a^{2/3}}{b^{1/2}}\right)^2 \left(\frac{b^{3/2}}{a^{1/2}}\right)$

8. Use properties of exponents and logarithms to solve for x:

A. $5^{x+1} = 25$

B. $\frac{1}{27} = 9^{2x+3}$

C. $\ln x^2 = 2\ln 4 - 4\ln 5$

9. Rewrite as the logarithm of a single quantity:

A. $\ln 5 + \ln(x^2 - 1) - \ln(x - 1)$

B. $\frac{3}{2}\left[\ln(x^2+1) - \ln(x+1) - \ln\left(x-1\right)\right]$

10. Simplify:

A. $e^{2\ln 5}$

B. $\ln\sqrt{e}$

C. $2\ln\sqrt{x} + 3\ln x^{1/3}$

11. Solve for the indicated variable (**no complex fractions** please):

A. $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1, \text{ for } a$

B. $2x-2yd=y+xd, \text{ for } d$

C. $\frac{2x}{4\pi}+\frac{1-x}{2}=0, \text{ for } x$

D. $A=2\pi r^2+2\pi rh, \text{ for } h$

12. Complete the square to write the equation of a parabola in $y=a(x-h)^2+k$ form or the equation of a circle in $(x-h)^2+(y-k)^2=r^2$. Identify the vertex of the parabola or identify the center and radius of the circle.

A. $3x^2+3x+2y=0$

B. $9x^2-6x-9-y=0$

C. $4x^2+4y^2-4x+24y+1=0$

13. Factor completely then solve the equation (real and imaginary roots):

A. $x^6-16x^4=0$

B. $8x^3+27=0$

C. $4x^3-8x^2-25x+50=0$

14. Evaluate without a calculator (of course!):

A. $\cos\frac{7\pi}{6}$ B. $\sin\frac{5\pi}{4}$ C. $\tan\frac{5\pi}{3}$ D. $\sin\frac{3\pi}{2}$

E. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ F. $\cos^{-1}\left(-\frac{1}{2}\right)$ G. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ H. $\cos^{-1}\left(\sin\left(-\frac{\pi}{4}\right)\right)$

15. Solve the equation in the indicated interval:

A. $3\cos^2 x = \sin^2 x; \quad 0 \le x < 2\pi$

B. $\sin^2 x - \cos^2 x = \sin x\,; \quad 0 \le x < 2\pi$

C. $\tan x + \sec x = 2\cos x; \quad -\infty < x < \infty$

16. Match the equation with its graph:

A. $y = \sin(x - \frac{\pi}{4})$

B. $y = \sin(\frac{1}{2}x)$

C. $y = \sin 2x$

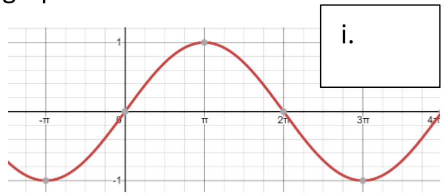
D. $y = 2\sin x$

E. $y = \cos x$

F. $y = \dfrac{1}{\sin x}$

G. $y = \sec x$

H.



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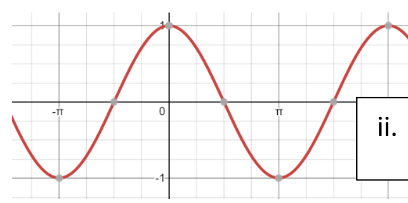

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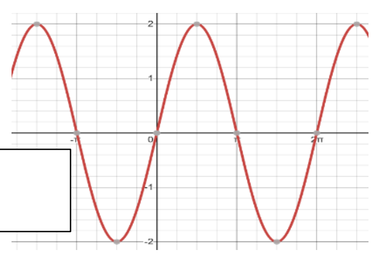

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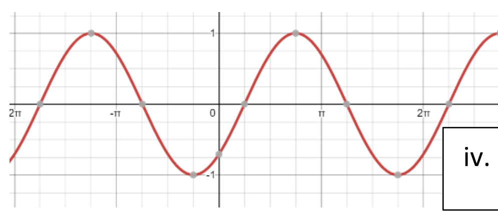

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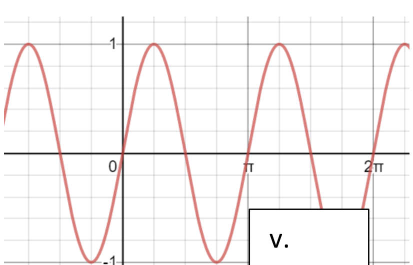

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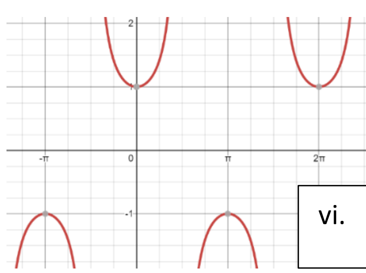

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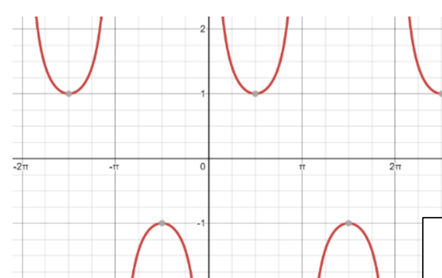


vii.

17. Find the remainders upon division of:

A. $(x^5 - 4x^4 + x^3 - 7x + 1)$ by $(x+2)$

B. $(x^5 - x^4 + x^3 + 2x^2 - x + 4)$ by $(x^3 + 1)$

18. Use factoring and the zero-product property to solve the equations:

A. $12x^3 - 23x^2 - 3x + 2 = 0$, given one solution is $x = 2$

B. $12x^3 + 8x^2 - x - 1 = 0$, given that all solutions are rational and between ± 1

19. Solve the inequalities. Use interval notation for the solutions. Hint: domain!

A. $\dfrac{2x-1}{3x-2} \le 1$

B. $\dfrac{1}{2x+3} > \dfrac{1}{x-5}$

C. $|4 - x| \le 1$

D. $|2x+1| > 3$

20. Write the equation of the following:

A. A line passing through point (-1, 2) and perpendicular to the line $2x-3y+5=0$.

B. A line passing through point (2, 3) and the midpoint of the line segment from (-1, 4) to (3, 2).

C. A circle centered at (1, 2) that passes through the point (-2, -1).

D. A circle that passes through the origin, and has x-intercept (1, 0) and y-intercept (0, 2).

21. Determine the domain of $f(x)=\dfrac{3x+1}{\sqrt{x^2+x-2}}$

22. Determine the domain and the range of the following:

A. $g(x)=\dfrac{6x-2}{2x+1}$

B. $h(x)=\dfrac{3|x|}{x}$

23. Evaluate (and simplify) $\dfrac{f(x+h)-f(x)}{h}$ for each of the following:

A. $f(x)=3x^2-x+5$

B. $f(x) = \sqrt{2x+3}$

24. Find the inverse $f^{-1}(x)$ of each of the following (with domain restriction if necessary: think graphically):

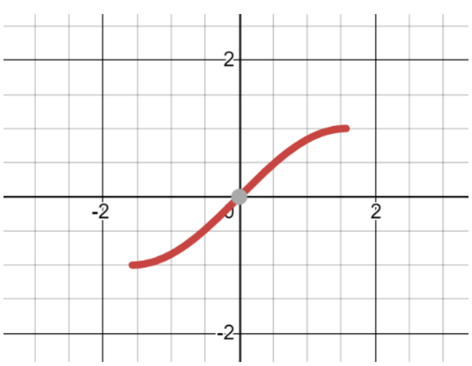
A. $f(x) = \sqrt{2x+3}$

B. $f(x) = \dfrac{x+2}{5x-1}$

C. $f(x) = x^2 - 6x + 10, \quad x > 3$

25. Sketch the inverse function and identify it by name (think trig!):

A.



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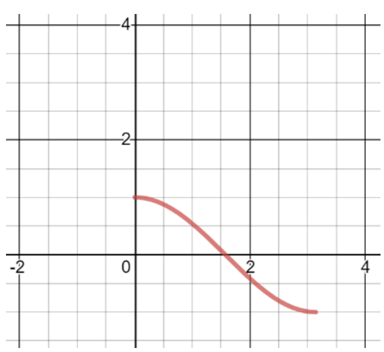

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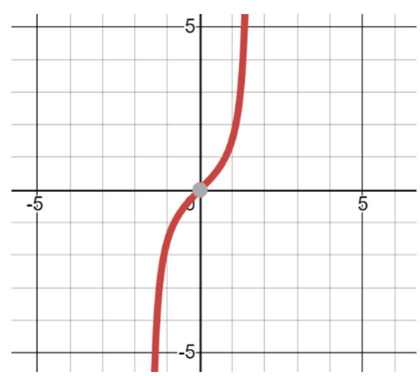
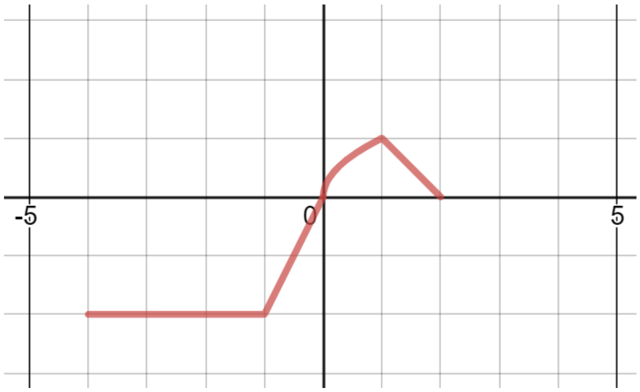


26. The graph of the function $f(x)$ is shown. Determine the graphs of the transformations.



A. $f(x+2)-1$

B. $f(-x)$

C. $|f(x)|$

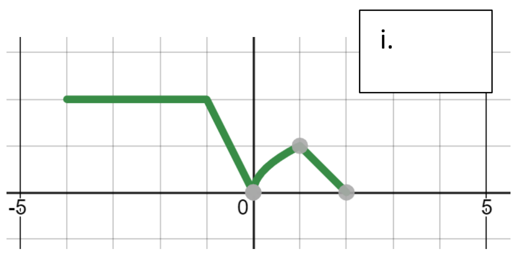
D. $f(|x|)$

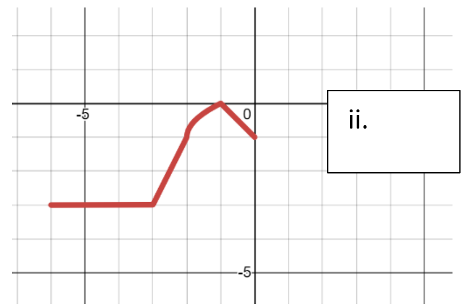
E. $f(|-x|)$

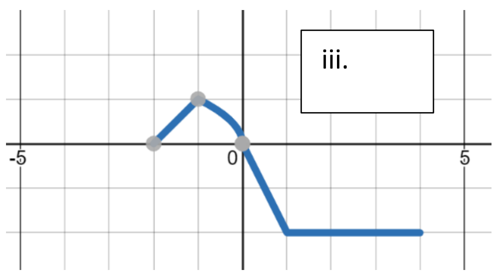
F. $2f(x)$

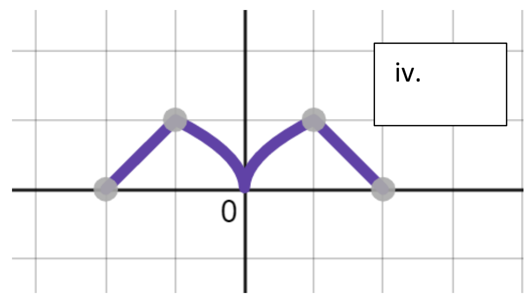
G. $f(2x)$

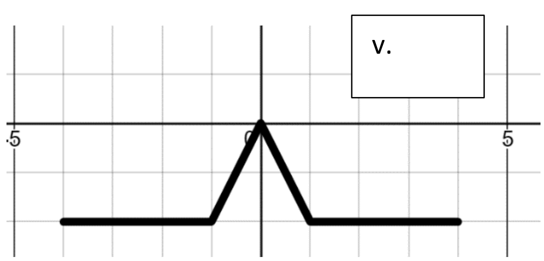
H. $f(\frac{1}{2}x)$

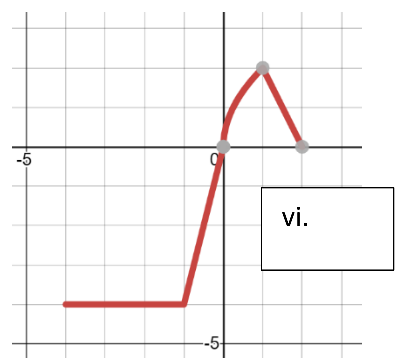


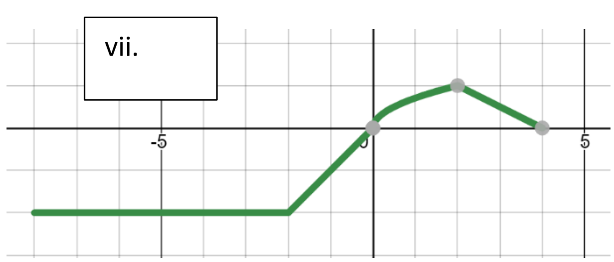


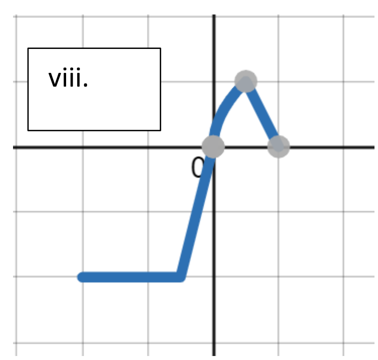






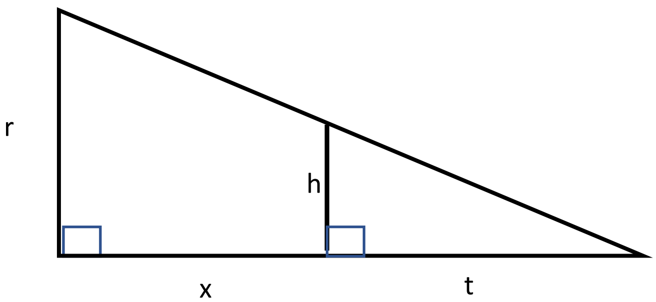




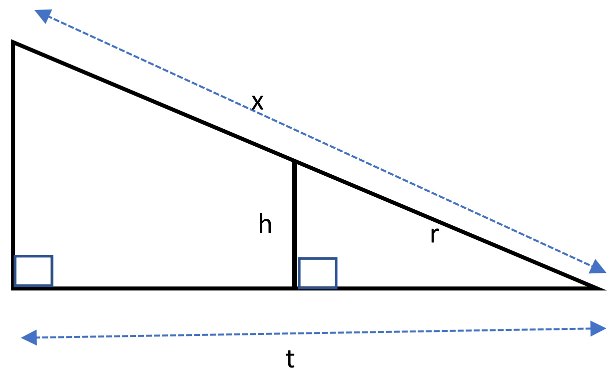


27. Use similar triangles to express x in terms of the other variables in the diagram:

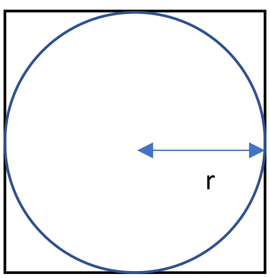
A.



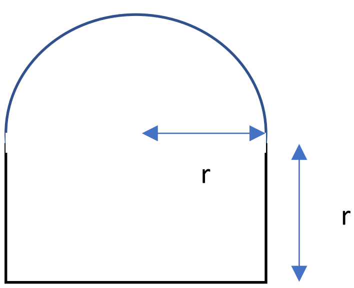
B



28. Find the ratio of the area inside the square but outside the circle to the area of the square:



29. Find an expression for the perimeter of the rectangle with a semicircle on top:



30. If $f(x)=2x-3$ and $g(x)=\sqrt{3x-1}$ evaluate the following:

A. $f\left(g(x)\right)$

B. $g\left(f(x)\right)$

31. If $f(x)=\dfrac{3}{x}$ and $g(x)=\dfrac{x}{2x-1}$, find $f\left(g(x)\right)$ and state its domain.