Directions: Complete the following problems on separate paper to turn in after coming back to school in the Fall.

1. Simplify: (a) $\dfrac{x^3 - 9x}{x^2 - 7x + 12}$ (b) $\dfrac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$ (c) $\dfrac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}}$ (d) $\dfrac{9 - x^{-2}}{3 + x^{-1}}$

2. Rationalize the denominator: (a) $\dfrac{2}{\sqrt{3} + \sqrt{2}}$ (b) $\dfrac{4}{1 - \sqrt{5}}$ (c) $\dfrac{1}{1 + \sqrt{3} - \sqrt{5}}$

3. Write each of the following expressions in the form $ca^p b^q$ where c, p and q are numbers:
   (a) $\dfrac{(2a^2)^3}{b}$ (b) $\sqrt{9ab^3}$ (c) $\dfrac{a(2/b)}{3/a}$ (d) $\dfrac{ab - a}{b^2 - b}$ (e) $\dfrac{a^{-1}}{(b^{-1})\sqrt{a}}$ (f) $\left(\dfrac{a^{2/3}}{b^{1/2}}\right)^2 \left(\dfrac{b^{3/2}}{a^{1/2}}\right)$

4. Solve for x (do not use a calculator):
   (a) $5^{(x+1)} = 25$ (b) $\dfrac{1}{3} = 3^{2x+2}$ (c) $\log_2 x = 3$ (d) $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

5. Simplify: (a) $\log_2 5 + \log_2(x^2 - 1) - \log_2(x - 1)$ (b) $2\log_4 9 - \log_2 3$ (c) $3^{2\log_3 5}$

6. Simplify: (a) $\log_{10}\left(10^{1/2}\right)$ (b) $\log_{10}\left(\frac{1}{10^x}\right)$ (c) $2\log_{10}\sqrt{x} + 3\log_{10} x^{1/3}$

7. Solve the following equations for the indicated variables:
   (a) $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$, for a (b) $V = 2(ab + bc + ca)$, for a
   (c) $A = 2\pi r^2 + 2\pi rh$, for positive r (d) $A = P + nrP$, for P
   (e) $2x - 2yd = y + xd$, for d (f) $\dfrac{2x}{4\pi} + \dfrac{1 - x}{2} = 0$, for x

8. For the equations   (a) $y = x^2 + 4x + 3$ (b) $3x^2 + 3x + 2y = 0$ (c) $9y^2 - 6y - 9 - x = 0$
   complete the square and reduce to one of the standard forms $y - b = A(x - a)^2$ or $x - a = A(y - b)^2$.

9. Factor completely: (a) $x^6 - 16x^4$ (b) $4x^3 - 8x^2 - 25x + 50$ (c) $8x^3 + 27$ (d) $x^4 - 1$

10. Find all real solutions to: (a) $x^6 - 16x^4 = 0$ (b) $4x^3 - 8x^2 - 25x + 50 = 0$ (c) $8x^3 + 27 = 0$

11. Solve for x: (a) $3\sin^2 x = \cos^2 x$; $0 \le x < 2\pi$ (b) $\cos^2 x - \sin^2 x = \sin x$; $-\pi < x \le \pi$
    (c) $\tan x + \sec x = 2\cos x$; $-\infty < x < \infty$

12. Without using a calculator, evaluate the following:
    (a) $\cos 210°$ (b) $\sin \dfrac{5\pi}{4}$ (c) $\tan^{-1}(-1)$ (d) $\sin^{-1}(-1)$
    (e) $\cos \dfrac{9\pi}{4}$ (f) $\sin^{-1} \dfrac{\sqrt{3}}{2}$ (g) $\tan \dfrac{7\pi}{6}$ (h) $\cos^{-1}(-1)$

13. Given the graph of $\sin x$, sketch the graphs of:
    (a) $\sin\left(x - \dfrac{\pi}{4}\right)$ (b) $\sin\left(\dfrac{x}{2}\right)$ (c) $2\sin x$ (d) $\cos x$ (e) $\dfrac{1}{\sin x}$

14. Solve the equations: (a) $4x^2 + 12x + 3 = 0$ (b) $2x + 1 = \dfrac{5}{x + 2}$ (c) $\dfrac{x + 1}{x} - \dfrac{x}{x + 1} = 0$

15. Find the remainders on division of:
    (a) $x^5 - 4x^4 + x^3 - 7x + 1$ by $x + 2$.   (b) $x^5 - x^4 + x^3 + 2x^2 - x + 4$ by $x^3 + 1$.
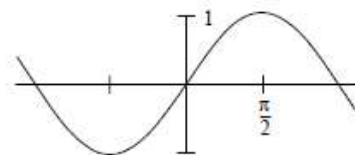
16. (a) The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution $x = 2$. Find all other solutions.
    (b) Solve for x, the equation $12x^3 + 8x^2 - x - 1 = 0$. (All solutions are rational and between ± 1.)

17. Solve the inequalities (a) $x^2 + 2x - 3 \le 0$ (b) $\dfrac{2x-1}{3x-2} \le 1$ (c) $x^2 + x + 1 > 0$

18. Solve for x: (a) $|-x+4| \le 1$ (b) $|5x - 2| = 8$ (c) $|2x + 1| = x + 3$

19. Determine the equations of the following lines: (a) the line through (-1,3) and (2,-4);
    (b) the line through $(-1, 2)$ and perpendicular to the line $2x - 3y + 5 = 0$;
    (c) the line through $(2, 3)$ and the midpoint of the line segment from $(-1, 4)$ to $(3, 2)$.

20. (a) Find the point of intersection of the lines: $3x - y - 7 = 0$ and $x + 5y + 3 = 0$
    (b) Shade the region in the $x - y$ plane that is described by the inequalities $\begin{cases} 3x - y - 7 & < & 0 \\ x + 5y + 3 & \ge & 0 \end{cases}$.

21. Find the equations of the following circles:
    (a) the circle with centre at $(1, 2)$ that passes through the point $(-2, -1)$;
    (b) the circle that passes through the origin and has intercepts equal to 1 and 2 on the $x-$ and
        $y-$ axes, respectively.

22. For the circle $x^2 + y^2 + 6x - 4y + 3 = 0$, find:
    (a) the centre and radius;   (b) the equation of the tangent at $(-2, 5)$

23. A circle is tangent to the $y-$axis at $y = 3$ and has one $x-$intercept at $x = 1$.
    (a) Determine the other $x-$intercept.   (b) Deduce the equation of the circle.

24. A curve is traced by a point $P(x, y)$ which moves such that its distance from the point $A(-1, 1)$ is
    three times its distance from the point $B(2, -1)$. Determine the equation of the curve.

25. (a) Find the domain of the function $f(x) = \dfrac{3x + 1}{\sqrt{x^2 + x - 2}}$.
    (b) Find the domain and range of the functions: i) $f(x) = 7$ ii) $g(x) = \dfrac{5x - 3}{2x + 1}$

26. Let $f(x) = \dfrac{|x|}{x}$. Show that $f(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$. Find the domain and range of $f(x)$.

27. Simplify $\dfrac{f(x + h) - f(x)}{h}$, where (a) $f(x) = 2x + 3$ (b) $f(x) = \dfrac{1}{x + 1}$ (c) $f(x) = x^2$.

28. The graph of the function $y = f(x)$ is given as follows:

    Determine the graphs of the functions:

    (a) $f(x + 1)$ (b) $f(-x)$ (c) $|f(x)|$ (d) $f(|x|)$

29. Sketch the graphs of the functions: (a) $g(x) = |3x + 2|$ (b) $h(x) = |x(x - 1)|$

30. (a) The graph of a quadratic function (a parabola) has $x-$intercepts -1 and 3 and a range consisting
        of all numbers less than or equal to 4. Determine an expression for the function.
    (b) Sketch the graph of the quadratic function $y = 2x^2 - 4x + 3$.